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The Impact of Population Growth on Investment, Income and Employment

Introduction

THE object of this paper is to analytically trace the long-term effect of population increase on investment and income through a traditional Harrod Domer growth model by explicitly introducing the population into the consumption function. A numerical demonstration of the new model is also given in the concluding section of this paper.

Growth models of the type made familiar by Harrod and Domer consist of the following sets of equations :

$$Y(t) = C(t) + I(t)$$
$$C(t) = K + Y(t)$$
$$Y(t) - Y(t-1) = 8I(t-1)$$

where $Y(t)$ is income, CO) consumption, 70) investment at time t. This is eventually reduced into a difference equation generating a time sequence of the type 70) = c70 — 1) or $Y(t\} = II \cdot Y(t - 1)$ leading to a solution in the form $I(t) = I_0 a^t$ or $Y(t) = Y_o II^t$

The model as stated above does not take any explicit cognisance of the population effect on consumption except through the aggregate consumption and income. *In an economy where the subsistence element of consumption is* very large, much of the increase in consumption may be due to a growth of the population itself and population as an explanatory factor would show the effect clearly. We shall, therefore, prepare first a separation of the total change in consumption into two parts : a part due to changes in population and a part due to changes in income. Separation of this type has been attempted previously by Stone in his linear expenditure systems, where he considers the expenditure as consisting of a part independent of income and a part dependent on income.

2. The Reformulated Model

Let $C(t)$ denote total consumption, $C_s(t)$ the consumption defining per capita *subsistence level which is independent of income and* C_v *the consumption* dependent on income.

Then, $$C(t) = C_s(t) + C_v(t).$$

Let the per capita subsistence level be defined by β. Then the consumption C_s is purely determined as a function of the population e.g.

$$C_s = \beta \cdot P(t)$$

where $P(t)$ is the population at t. After supporting the population at the subsistence level, therefore, the income remaining Y_r is seen to be

$$\begin{aligned} Y_r &= Y(t) - C_s(t) \\ &= Y(t) - \beta \cdot P(t). \end{aligned}$$

We can now *say that the consumption induced by income is a function of the* residual income e.g.

$$C_v = K + \delta\,(Y(t) - \beta\, P(t)).$$

Then we have,

$$\begin{aligned} C_t &= C_s(t) + C_v(t) \\ &= \beta \cdot P(t) + K + \delta\,(Y(t) - \beta\, P(t)) \\ &= K + \delta Y(t) + \beta\,(1 - \delta)\, P(t) \end{aligned}$$

The above equation may be rewritten as

$$C(t) = K + \alpha \cdot Y(t) + \beta \cdot P(t).$$

The Harrod-Domer system now is therefore reformulated as below :

$$Y(t) = C(t) + I(t)$$
$$C_t = K + \alpha \cdot Y(t) + \beta \cdot P(t)$$
$$Y(t) - Y(t-1) = \delta \cdot I(t-1) + M$$

In the reformulated model the population effect is now explicitly **introduced.** An expansion of the sequence of income and investment over time reveals the separate effect of population itself.

Solving the above system for a one period process we get,

$$Y(t) = (1 + \delta - \delta \cdot \alpha)\ Y(t-1) - \delta \cdot \beta \cdot P(t-1) + (M - \delta \cdot K)$$
$$= [1 + \delta\,(1-\alpha)]\ Y(t-1) - \delta \cdot \beta \cdot P(t-1) + (M - \delta \cdot K)$$

$(1-\alpha)$ in the above system is the saving coefficient and, therefore, the first part in effect shows the effect of the saving rate on growth, while the second part shows the cut back on expansion of income due to increased pressure of subsistence level itself on total consumption.

Considered as a process over θ years we get the sequence as follows :

$$Y_t = (1 + \delta - \delta\,\alpha)^{\theta}\ Y(t-\theta) - (1 + \delta - \delta\alpha)^{\theta-1} \cdot \alpha\beta P(t-\theta)$$
$$- (1 - \delta - \delta \cdot \alpha)^{\theta-2}\ \delta\beta P(t-\theta-1) - (1 + \delta - \delta\alpha)\ \delta\beta P(t-2)$$
$$- \delta\beta P(t-1) + (1 + \delta - \delta\alpha)^{\theta-1}\ (M - \delta K)$$
$$+ (1 + \delta - \delta\alpha)^{\theta-2}\ (M - \delta K) + (M - \delta K)$$

Similarly, we get a one year growth process for investment as below :

$$I(t) = (1-\alpha)\ (1 + \delta - \delta\alpha)^{\theta} \cdot Y(t-1) - (1-\alpha)\ \delta\beta P(t-1)$$
$$- \beta\ P(t) + (1-\alpha)\ (M - \delta K) - K.$$

Extending the process to θ years we have,

$$I(t) = (1-\alpha)(1+\delta-\delta\alpha)\ Y(t-\theta) - (1-\alpha)(1+\delta-\delta\alpha)^{\theta-1}\beta P(t-2)$$
$$-(1-\alpha)(1+\delta-\delta\alpha)^{\theta-2}\beta P(t-\theta-1) - (1-\alpha)\delta\beta P(t-1) - \beta P(t)$$
$$+(1-\alpha)(1+\delta-\delta\alpha)^{\theta-1}(M-\delta K) - (1-\alpha)(1+\delta-\delta\alpha)^{\theta-2}(M-\delta K)$$
$$(1-\alpha)(M-\delta K) - K.$$

The growth process worked back from θ years can be expressed in more convenient form as below :

$$Y(t) = [1+\delta(1-\alpha)]^{\theta}\, Y(t-1) + \sum_{\theta=1}^{\theta} [1+\delta(1-\alpha)]^{\theta-1}\delta\beta P(t-\theta) + \text{const.}$$

and

$$I(t) = (1-\alpha)[1+\delta(1-\alpha)]^{\theta} - \sum_{\theta=1}^{\theta} [(1-\alpha)\{1+\delta(1-\alpha)\}]^{\theta-1}\delta\beta P(t-\theta)$$
$$-\beta P(t) + \text{const.}$$

The expressions under the negative sign in the two show the sequence of deductions in $Y(t)$ and $I(t)$ due to a population effect starting θ years hence. This is also obvious from commonsense. A birth taking place 10 years back say, has been causing a sequence of cut backs on the savings due to increased population and the effects are cumulated over the whole interval to get the full effect over ten years.

3. An Empirical Demonstration

We shall now demonstrate an empirical model which was fitted to Indian data. The following constants were fitted to the model by the Method of Least Square :

$$C(t) = -4425.2862 + 0.9494\ Y + 0.00818\ P$$

$$Y(t) = 9538.1102 + 0.2955 \sum_{K=0}^{t-1} I_k.$$

The investment function was fitted in this cumulated data as fitting with first difference led to very poor results. The cumulative fitting process, however, gives a reasonable marginal capital/output ratio.

We give below the projection for income and investment which were computed with 0 = 1 and 6 = 5 respectively from growth model for income and investment respectively.

TABLE 1

(Unit = 10 million Rs.)

	Observed	*Estimated from a process of I year*	*Estimated from a process over 5 years*
Income (1967)	16456	16548	16662
Income (1962)	14623	14443	12850
Investment (1967)	2615	5261	5256
Investment (1962)	2165	5154	5066

It may be seen that both income and investment are reasonably well projected. The present investment function, however, does not include a possible effect of increased labour supply on income.

We now add to the model two further equations relating investment to employment and labour force to the population :

$$E(t) - E(t-1) - - 0.0123 + 0.0000229\ I(t \quad 1)$$
$$L(t) - L(t-1) = + 0.0635\ [P(t) - P(t-1)]3,$$

where $E(t)$ is employment and $L(t)$ the labour force. The data were fitted with two year averages to smooth out irregularities. Labour force has been fixed on the basis of the ratio of labour to population in the 1961 Census.

The following figures were obtained as estimates of the labour force and of employment by this model. Projecting the labour force as a constant fraction of the population based on Census of 1951 and 1961, a gap between projected employment and the labour force looking for employment were computed as follows :

TABLE 2

(Unit = 10 million)

	1967 (projected from data of 1962)	*1962 (projected from data of 1961)*
Labour force	4.67	4.27
Employment	0.1409	0.1354

Assuming that the need of population first to feed itself is dominant, the impact of population on investment on the one hand and on labour force on the other becomes patently clear. While projection of labour has grown by 14 million, projected employment has grown by 0.05 million.

4. The Positive Effect of Population on Income

The above model, however, is rather one-sided in the sense that productivity of labour is taken to be constant and output is not in any way assumed to be affected by the labour force. But if we assume that with growth of population income increases to some extent, we can modify the income equation as follows :

$$Y(t) - Y(t-1) = \delta_1 + \delta_2\,[P(t) - P(t-1)] + \delta_3\, I(t-1).$$

This is analogous to the Cobb-Douglas type in a linear form i.e.

$$Y = A \cdot K^{\delta_1} L^{\delta_2}$$

or

$$\frac{\Delta Y}{Y} = \text{const.} + \delta_1 \left[\frac{\Delta K}{K}\right] + \delta_2 \left[\frac{\Delta L}{L}\right] = \text{const.} + \delta_1 \left[\frac{\Delta K}{K}\right] + \delta_2 \left[\frac{\Delta P}{P}\right]$$

if $$\frac{\Delta L}{L} = \frac{\Delta P}{P}.$$

Instead however of proportionate change we are considering a simple linear change only in the form $Y = \delta_1 + \delta_2\, K + \delta_3 \cdot P$.

It must be explained that this form is taken simply with a view to demonstrate growth over time in a simple way by bringing in the two-sided effect of population growth on savings (by reducing savings) and on output by employing more labour (as marginal productivity of labour is > 0). Under such a condition, the sequence will naturally take a more cumbrous form as given below :

$$\begin{aligned} Y(t) = {} & (1 + \delta_3 - \delta_3\alpha)^{\theta}\, Y(t-\theta) + \delta_2\, P(t) + \delta_2\,(1 + \delta_3 - \delta_3\alpha)\, P(t-\theta) \\ & + \ldots + \delta_2\,(1 + \delta_3 - \delta_3\alpha)^{\theta}\, P(t-\theta) - (\delta_2 + \delta_3\beta)\, P(t-1) \\ & - (1 + \delta_3 - \delta_3\alpha)(\delta_2 + \delta_3\beta)\, P(t-2). \end{aligned}$$

Assuming an exponential growth of population, we get

$$Y(t) = (1 + \delta_3 - \delta_3\alpha)^{\theta} \; Y(t - \theta) + \delta_2 \, P(t - \theta) \sum_{j} (1 + \delta_3 - \delta_3\alpha)^{\theta - j} \, e^{(j-1)}$$

$$- (\delta_2 + \delta_3) \, P(t - \theta) \; \Sigma \; (1 + \delta_3 - \delta_3\alpha)^{\theta - j} \, e^{\varepsilon(j+1)} + \text{const.}$$

It may be appreciated that no unconditional statement can any longer be made about the output due to increase in population. The final result depends on both the marginal consumption coefficient for the population as also the marginal productivity coefficient for labour.